Forward Error Correction of FSK Alphabets for Noncoherent Transmissions over AWGN Channel

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***Abstract*—We show that several coding schemes, in particular turbo encoding, dramatically improve the performance of a noncoherently detected frequency shift-keying modulation with correlated signals.The proposed *a posteriori* probability (APP) decoder based on bit-by-bit decisions proves to be highly flexible and more efficient than a classical block decisions decoding scheme. Moreover, the choice of the FSK modulation tone spacing is important. Simulation results show that the turbo-coded FSK exhibits the lowest error probability when compared to TCM, convolutional and lattice encoding schemes.**

I. Introduction

FREQUENCY shift-keying (FSK) modulation with orthogonal signals is commonly used on noncoherent channels and exhibits an excellent robustness [1]. However, it suffers from its poor spectral efficiency. With a modulation tone spacing chosen smaller than $1/T$, T being the symbol period, the transmitted signals are correlated and the spectral efficiency increases. The price to pay for this bandwidth reduction is a degradation in the noncoherent detector performance.It can be shown that several coding schemes dramatically reduce this performance degradation.

We describe two system models. The decoder of the first model makes classical block decisions from the signal correlations delivered by the matched filter bank. In the second model, the FSK noncoherent detector computes APP's for each coded bit, directly related to these correlation values. The decoder operates on these APP's, considered as channel observations, to make bit-by-bit decisions. Note that the second model is valid for all coding schemes: simple block or convolutional codes, parallel or serial concatenated (turbo) codes [2] and classical trellis-coded modulations (TCM) [3].

Computer simulation results show that the turbo-coded scheme yields better results than either TCM, convolutional or lattice encoded schemes. As expected high latency coding schemes perform better than low latency schemes.

II. Multidimensional Noncoherent Transmisssions

Consider a Q-ary frequency shift-keying modulation with tone spacing and symbol period respectively denoted by Δf_0 and T.

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A block of K information bits, denoted $b_1, b_2, \cdots, b_K$, are encoded and mapped to N integer components p_n, $n = 1, \cdots, N$, each belonging to the Q-ary set $\{\pm 1, \pm 3, \cdots, \pm(Q-1)\}$. Note that $K = R \times N \times \log_2(Q)$ where R is the coding rate. These components are fed to the Q-FSK modulator: we associate an integer p_n at the encoder output to an elementary signal $s_n(t) = \sqrt{1/T}\, e^{j2\pi f_n t}$, for $0 \le t < T$ and $1 \le n \le N$. The elementary frequency f_n takes Q possible values uniformly spaced by Δf_0. f_n is proportional to the nth integer component p_n, $n = 1, \cdots, N$: $f_n = p_n \times (\Delta f_0/2)$. Hence, $s_n(t)$ belongs to $\{s_1(t), \cdots, s_Q(t)\}$.

The memoryless channel is characterized by a complex additive white Gaussian noise $b_n(t)$ with power spectral density $2N_0$ (i.e. variance $\sigma_b^2 = N_0$ for each component) and a random phase ϕ_n that is unknown to the receiver. The random variables ϕ_n are uniformly distributed, i.e. $p(\phi_n) = (1/2\pi)$.

Assuming that the nth transmitted signal is $s_n(t)$, the received signal can be written $r_n(t) = e^{j\phi_n} s_n(t) + b_n(t)$. When signals are extracted from an alphabet of M N-dimensional points $\{S_m = (s_{m,1}, \cdots, s_{m,N})\}_{m=1,\cdots,M}$, with $s_{m,n}(t) = \sqrt{1/T} e^{j2\pi f_{m,n} t}$, for $(n-1)T \le t < nT$, $1 \le m \le M$ and $1 \le n \le N$, it is possible to derive the optimum receiver which is given by the following criterion

$$\max_{m=1}^{M} \prod_{n=1}^{N} I_0 \left(\frac{1}{N_0} \left| \int_{(n-1)T}^{nT} r_n(t) s_{m,n}^*(t)\, dt \right| \right) \qquad (1)$$

$I_0(x)$ is the modified Bessel function of order zero.We denote $x_{m,n} = \int_T r_n(t) s_{m,n}^*(t)\, dt$. The set $\{x_{m,n}\}_{n=1,\cdots,N}$ is obtained by passing the received signals $r_n(t)$ through a bank of Q filters matched to $\{s_{m,n}(t)\}_{n=1,\cdots,N}$, and sampling their outputs at time $t = T$. The Maximum-Likelihood (ML) criterion is therefore directly derived from the matched filter bank outputs.

For low signal-to-noise ratios, $\log I_0(x)$ can be approximated by $x^2/4$, leading to the equivalent criterion

$$\max_{m=1}^{M} \sum_{n=1}^{N} \left| \int_{(n-1)T}^{nT} r_n(t) s_{m,n}^*(t)\, dt \right|^2 \qquad (2)$$

Our computer simulations as those of [4] confirmed that using this approximation yields a negligible performance degradation.

It can be noticed that the set $\{x_{m,n}\}_{n=1,\cdots,N}$ is redundant. We denote $x_{q,n} = \int_T r_n(t) s_q^*(t)\, dt$ the signal correlation at the output of the qth matched filter and $X_n = (|x_{0,n}|, |x_{1,n}|, \cdots, |x_{Q-1,n}|)$ the global observation for component n. The N vectors X_n constitute a sufficient statistics to make a decision. An exhaustive search based on (2) yields a high complexity decoding scheme. That's why we focus on nonoptimal decoders.

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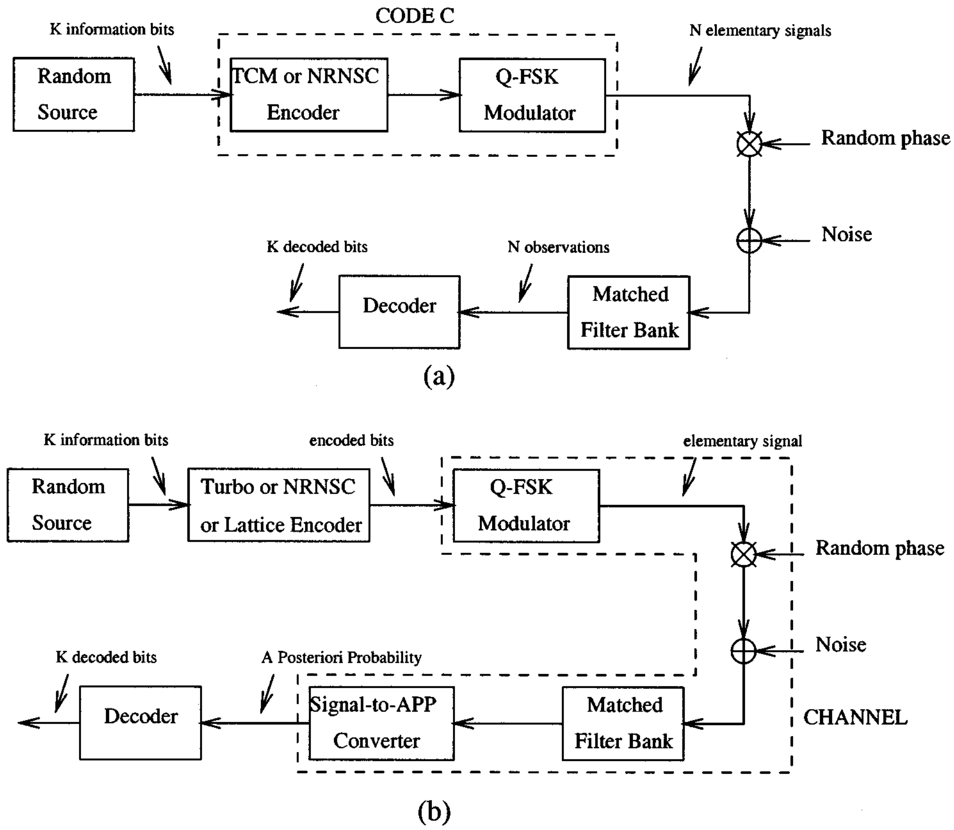

Fig. 1. (a) Encoded FSK alphabet and ML decoding. (b) Encoded FSK alphabet and APP decoding.

III. Blockwise MAP Decoder vs. APP Detector

In the first system model Fig. 1(a), the integer components p_n are gathered to form an N-dimensional point, denoted $P_m = (p_{m,1}, \cdots, p_{m,N})$, associated—through the modulator—to $S_m = (s_{m,1}, \cdots, s_{m,N})$, $m = 1, \cdots, M$.The concatenation of the encoder and the FSK modulator constitutes an overall code denoted by C. The C encoder delivers a signal of length N belonging to a multi-dimensional encoded FSK alphabet of $M = 2^K$ signals $S_m, m = 1, \cdots, M$.

After matched filtering, the decoder uses a blockwise maximum-*a-posteriori* (MAP) criterion, equivalent to ML decoding when signals have equal *a priori* probabilities. The decoding is directly based on equation (2). For example, this criterion may be used for the metric calculation in a Viterbi algorithm to decode a TCM or a Non-Recursive Non-Systematic Convolutional (NRNSC) code.

The second system model is given in Fig. 1(b). For each time interval $[(n-1)T, nT[$, the observation X_n is directly used by the signal-to-APP converter. This converter derives one APP for each coded bit, computed from the APP of the transmitted signal $s_n(t)$. As shown in Fig. 1(b), the concatenation of the Q-FSK modulation/demodulation and the Gaussian channel can be considered as a global binary input/soft output channel. The bitwise decision made by the decoder is based on the set of soft observations $\{X_n\}_{n=1,\cdots,N}$ delivered by this channel.

If no *a priori* information is available, a simple application of Bayes rule proves that the observation given by p(X_n/coded bit $c_{j,n}$), $j = 1, \cdots, \log_2(Q)$ at the channel output is proportional to the *a posteriori* probability given by P(coded bit $c_{j,n}/X_n$).

These values will not be distinguished in the sequel. Therefore, we compute a posteriori probabilities at the channel output calling this operation signal-to-APP conversion. These APP's will be considered as input observations by the decoder which computes new values of APP's considering both channel information and code constraints to make final decisions.

The observation for bit $c_{j,n}$ can be written

$$p(X_n/c_{j,n}) = p(|x_{0,n}|, |x_{1,n}|, \cdots, |x_{Q-1,n}|/c_{j,n}) = \sum_{q=0}^{Q-1} p(X_n/q, c_{j,n}) p(q/c_{j,n}) \qquad (3)$$

where $p(q/c_{j,n}) = 1$ if bit j of signal q equals $c_{j,n}$, and 0 otherwise.

Consequently, the signal-to-APP converter delivers a set of $\log_2(Q)$ *a posteriori* probabilities every time interval $[(n-1)T, nT[$ denoted by $p(c_{j,n}/X_n)$. Each APP $p(c_{j,n}/X_n)$ is proportional to

$$p(X_n/c_{j,n}) = \sum_{q/c_{j,n}} p(X_n/q) = \sum_{q/c_{j,n}} \text{APP}(q, n) \qquad (4)$$

where APP(q, n) is the *a posteriori* probability of signal q computed as described in [5].

IV. Simulation Results

All computer simulations are based on an 8-FSK modulation ($Q = 8$). Turbo code results were obtained with an interleaver

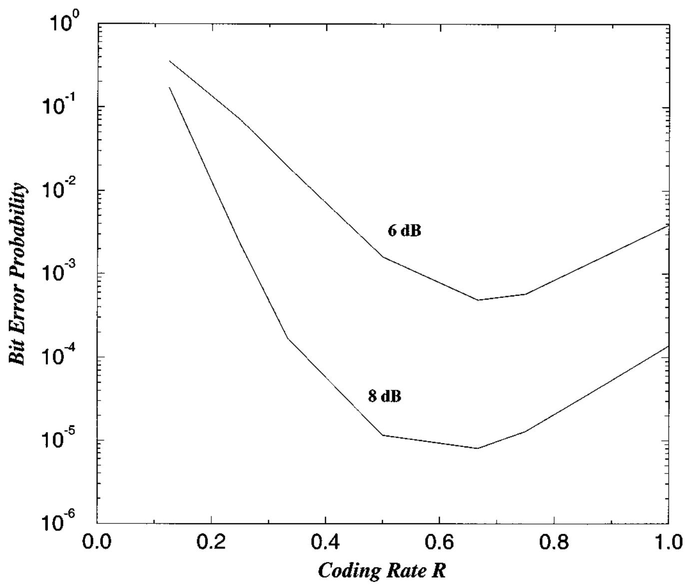

Fig. 2. Bit Error Probability vs. Coding rate $R = \Delta f_0 T$.

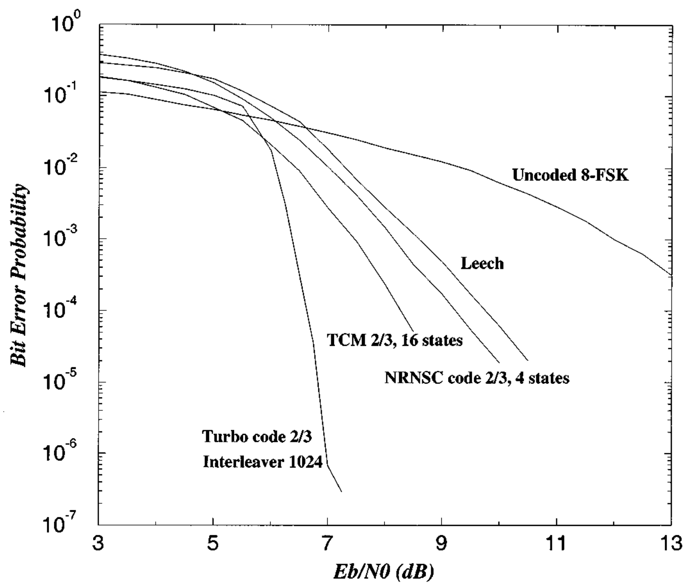

Fig. 3. Uncoded 8-FSK modulation, Leech lattice encoded 8-FSK, NRNSC coded, TCM 2/3 and Turbo-coded 8-FSK modulation (rate 2/3). Tone spacing $\Delta f_0 T = 0.34$.

of size 1024 and 12 decoding iterations (SISO iterative decoder based on a Forward Backward algorithm).This turbo code is a classical parallel concatenation of two RSC's with octal generators 23, 35. The NRNSC code in Fig. 3 has rate 2/3 and octal generators 27, 75, 72. Also, the trellis coded 8-PAM modulation used below has a rate 2/3 with octal generators 23, 04, 16.

In Fig. 2, the bit error probability of an encoded 8-FSK modulation is depicted as a function of the coding rate R, for two fixed values of the SNR, 6 and 8 dB. We considered NRNSC codes of comparable complexity with respective rate R and constraint length L (1/8, 5), (1/4, 5), (1/3, 5), (1/2, 5), (2/3, 3) and (3/4, 2). The tone spacing is chosen equal to the coding rate, $\Delta f_0 T = R$, so that all the considered schemes have the same spectral efficiency. As observed in Fig. 2, the optimal tone spacing is equal to $\Delta f_0 T \approx 2/3$ as the bit error rate is minimized independently from the signal-to-noise ratio value. Notice that the more we reduce the tone spacing, the less the code will be able to compensate for the degradation. Furthermore, when $\Delta f_0 T$ increases up to 1.0, the degradation is not prohibitive anymore, but the corresponding high rate codes are not very efficient. Hence, the choice of the tone spacing is of main interest.

Fig. 3 shows the performance of the uncoded 8-FSK modulation versus four coded schemes for $\Delta f_0 T = 0.34$: Λ_{24} lattice coded alphabets, 4-state NRNSC coded 8-FSK modulation (rate 2/3), 16-state trellis-coded 8-FSK modulation (rate 2/3), and Turbo-coded 8-FSK modulation (rate 2/3). The Leech lattice Λ_{24} is the densest lattice sphere packing in dimension 24 [6]. The spectral efficiency of the uncoded modulation is 1.125 bit/sec/Hz while for all coded schemes, it is 0.75 bit/s/Hz. Since the TCM mapping optimizes the distance between signals, the classical blockwise decoder of the first model yields better performance than the APP decoder for TCM coded FSK. However, in case of convolutional coding, the APP decoder associated to an interleaver performs better than the ML decoder, since the bit-by-bit decision technique avoids bursts of errors.

All coding schemes prove their better robustness to correlation than the uncoded modulation. The Turbo-coded 8-FSK exhibits the best results when compared to lattice encoded, convolutional encoded and TCM encoded alphabets. However, one should not forget to take into account the latency of the coded systems which is minimum for the Leech lattice and maximum for the turbo code. Finally, it is worth to note that the coding gain is achieved at the cost of a higher detection complexity, as encountered in all classical coded systems. Both lattice and trellis decoders used above have a comparable complexity and are less greedy than the turbo decoder.

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