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The Capture Effect in FM Receivers

KRIJN LEENTVAAR AND JAN H. FLINT

Abstract—In this paper a theoretical explanation of the capture effect is given by calculating the instantaneous frequency of the output signal of a limiter when two frequency modulated (FM) signals are present at the limiter input. When this signal is applied to a demodulator with unlimited bandwidth, the output signal of the demodulator proves to have an extreme capture effect. When however the demodulator bandwidth is limited, the capture effect is shown not be be extreme. This phenomenon is explained and possibilities are given to minimize the capture effect.

Some of the results of measurements on limiters and demodulators are given in this paper; they prove that a weak capture effect can be obtained. A method of calculating the degree of capturing is included.

INTRODUCTION

WHEN a frequency modulated (FM) receiver has two different FM signals with unequal amplitudes falling within the passband at the same time, the modulation of the weaker signal no longer exists at the demodulator output or at least is attenuated to a very high degree. This also appears when the stronger signal is unmodulated. This phenomenon is known as the capture effect.

In this paper, first the phasor diagram will be considered by which it is possible to calculate the output signal of a limiter and its instantaneous frequency when two FM signals are present at the limiter input. To illustrate the problem the frequency spectrum of the output signal is calculated. A function is given to express the mean frequency of the limiter output signal.

It is possible to explain the reduction of the capture effect by limiting the bandwidth of the demodulator.

A method of calculating these effects is given for a Foster-Seely demodulator.

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I. THE PHASOR DIAGRAM

Suppose the two different signals at the input of the limiter are a_1 and a_2. These signals are shown in Fig. 1. The signals may be expressed as

$$a_1 = A_1 \cos \phi_1 = \mathrm{Re}\,[A_1 e^{j\phi_1}] \tag{1}$$

$$a_2 = A_2 \cos \phi_2 = \mathrm{Re}\,[A_2 e^{j\phi_2}] \tag{2}$$

where

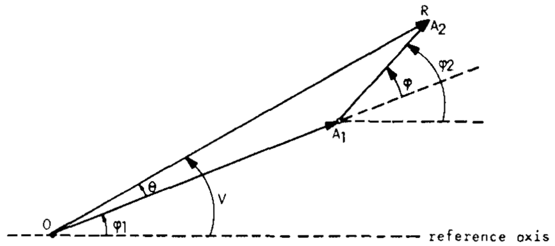

Fig. 1. Phasor diagram of two superimposed signals a_1 and a_2.

$$\phi_2 - \phi_1 = \phi \tag{3}$$

and

$$\frac{A_2}{A_1} = A. \tag{4}$$

ϕ_1 and ϕ_2 are the phase angles referred to the horizontal axis, and A_1 and A_2 are the amplitudes. We can define the radial frequency (or "frequency") of the signals as

$$\Omega_1 = \frac{d\phi_1}{dt} \tag{5}$$

$$\Omega_2 = \frac{d\phi_2}{dt} \tag{6}$$

and

$$\Omega = \Omega_2 - \Omega_1. \tag{7}$$

When the signals are phase modulated (PM) or FM, Ω_1 and Ω_2 will not be constant. Therefore, we call Ω_1 and Ω_2 the instantaneous frequencies of the signals, and ϕ_1 and ϕ_2 are the instantaneous phase angles.

The phasor diagram shows that if $A < 1$, and $A_2 < A_1$, the phase variation $\Delta\theta$ of the resultant R will be smaller than the phase variation $\Delta\phi$. As an illustration, Fig. 2 gives the ratio $\Delta\theta/\Delta\phi$ as a function of the amplitude ratio A, when $\Delta\phi$ is smaller than 90°.

If we suppose that a signal a_1 is unmodulated and a_2 is PM, while a_1 and a_2 have the same carrier frequency, $\Delta\theta$ is a measure of the modulation of the resulting signal with amplitude R. When this signal is supplied to a frequency demodulator, the output of this demodulator is proportional to the phase changes of R and also $\Delta\theta$. The output of the demodulator as a function of A will be the same as given in Fig. 2. This output is noticeably attenuated when $A < 10$. At $A = 1$ the attenuation proceeds smoothly and there is no capture effect at all.

The mean frequency of the resultant tends to follow the frequency of the stronger signal; when the frequencies of a_1 and a_2 are different but constant, the instantaneous frequency is *not* constant, but is a function of time. In this case the resultant R is amplitude modulated (AM) too. The modulations of R are not a simple harmonic.

This is very clear when $A_1 = A_2$. At the moment $\phi = \pi$, θ jumps from $+\pi/2$ to $-\pi/2$, the instantaneous frequency change will be infinite and the amplitude modulation of R will be 100 percent.

II. THE LIMITER

The amplitude of the resulting signal R from Fig. 1 is given by

$$R = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1A_2 \cos\phi}. \tag{8}$$

The phase angle ψ of the resultant is given by

$$\Psi = \phi_1 + \arctan\frac{A \sin\phi}{1 + A\cos\phi}. \tag{9}$$

The envelope of the resulting signal will have a maximum of $R = A_1 + A_2$ at $\phi = 0$ and a minimum of $R = A_1 - A_2$ at $\phi = \pi$. The time interval between the maxima and minima is determined by the frequency difference between signal a_1 and signal a_2.

If one of these signals or both are FM, these intervals are very variable.

To cancel amplitude variations at demodulation the limiter must deliver a signal with constant amplitude even at the moment $R = |A_1 - A_2|$. All time constants in the limiter circuit must be so chosen that this is assured at any frequency difference of a_1 and a_2. In the following we will suppose all these conditions are fulfilled. Therefore the output of the limiter is a signal of constant amplitude and a phase angle Ψ.

III. THE INSTANTANEOUS FREQUENCY OF THE LIMITER OUTPUT SIGNAL

The instantaneous frequency of the limiter output signal is given by

$$\Omega_r = \frac{d\Psi}{dt} = \frac{d\phi_1}{dt} + \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\,\frac{d\phi}{dt}$$
$$= \Omega_1 + \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\,\Omega \tag{10}$$

as can be found by differentiating (9).

This is the instantaneous frequency of the limiter output signal if two different signals are supplied to its input.

The phase angle

$$\Psi = \phi_1 + \arctan\frac{A\sin\phi}{1 + A\cos\phi} \tag{9}$$

can be expanded in a Taylor's series

$$\Psi = \phi_1 - \sum_{n=1}^{\infty} \frac{(-A)^n}{n} \sin n\phi. \tag{11}$$

If both signals are FM,

$$\Omega_1 = \omega_1 + \Delta\omega_1 \cos\mu_1 t \tag{12}$$

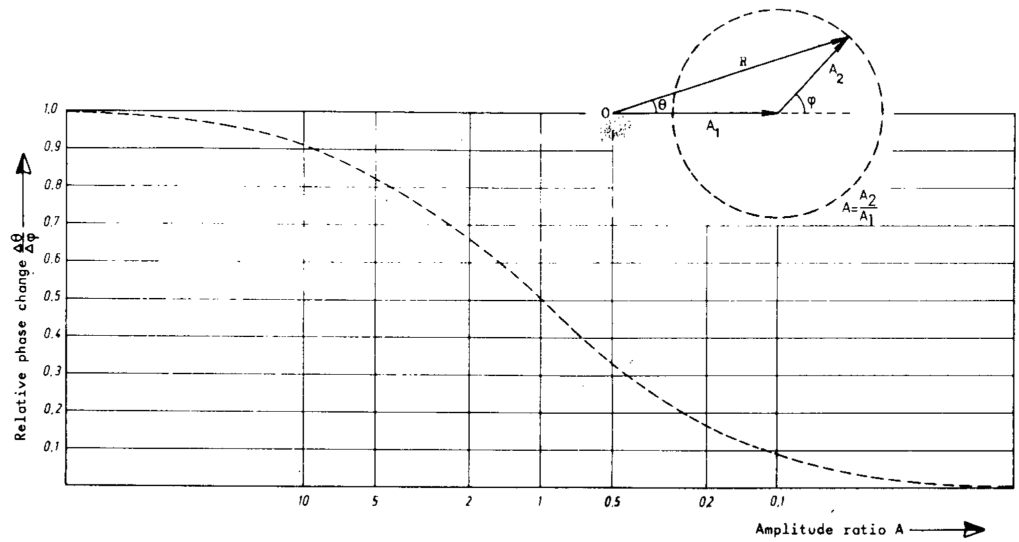

Fig. 2. Relative phase variation $\Delta\theta/\Delta\phi$ of the resultant R of two superimposed signals as a function of the amplitude ratio $A = A_2/A_1$, valid for $\Delta\phi < 90°$.

and

$$\Omega_2 = \omega_2 + \Delta\omega_2 \cos \mu_2 t, \tag{13}$$

then

$$\phi_1 = \omega_1 t - \frac{\Delta\omega_1}{\mu_1} \sin \mu_1 t = \omega_1 t - m_1 \sin \mu_1 t \tag{14}$$

and

$$\phi_2 = \omega_2 t - \frac{\Delta\omega_2}{\mu_2} \sin \mu_2 t = \omega_2 t - m_2 \sin \mu_2 t \tag{15}$$

$$\sin n\phi = I_m [e^{jn\phi}] = I_m [e^{jn(\phi_2 - \phi_1)}] = I_m [e^{jn(\omega_2 t - \omega_1 t)} e^{+jnm_1 \sin \mu_1 t} e^{-jnm_2 \sin \mu_2 t)}]. \tag{16}$$

$e^{jnm \sin \mu t}$ may also be expanded in a Taylor's series

$$e^{jnm_1 \sin \mu_1 t} = \sum_{p=-\infty}^{+\infty} I_p(um_1) e^{jp\mu_1 t} \tag{17}$$

and

$$e^{-jnm_2 \sin \mu_2 t} = \sum_{q=-\infty}^{+\infty} I_q(nm_2) e^{-jq\mu_2 t}. \tag{18}$$

So sin n can be written as

$$\sin n\phi = I_m \left[\sum_{p=\infty}^{\infty} I_p(nm_1) \sum_{q=\infty}^{+\infty} I_q(nm_2) \cdot e^{j(n\omega_2 t - n\omega_1 t - q\mu_2 t + p\mu_1 t)} \right]. \tag{19}$$

When we substitute this in (11) we find

$$\Psi = \phi_1 - \sum_{n=1}^{\infty} \frac{-(A)^n}{n} \sum_{p=\infty}^{+\infty} I_p(nm_1) \sum_{q=\infty}^{+\infty} I_q(nm_2) \cdot \sin (n\omega_2 t - n\omega_1 t - q\mu_2 t + p\mu_1 t). \tag{20}$$

The instantaneous frequency of the limiter output signal is then

$$\frac{d\Psi}{dt} = \omega_1 + \Delta\omega_1 \cos \mu_1 t - \sum_{n=1}^{+\infty} \frac{-(A)^n}{n} \sum_{p=-\infty}^{+\infty} I_q(nm_1) \cdot \sum_{q=-\infty}^{+\infty} I_q(nm_2)(n\omega_2 - n\omega_1 - q\mu_2 + p\mu_1) \cdot \cos (n\omega_2 t - n\omega_1 t - q\mu_2 t + p\mu_1 t). \tag{21}$$

Equation (15) gives the frequency of the signal at the output of a limiter when the input of this limiter consists of two different FM signals.

If this signal is supplied to a frequency demodulator, at the output of the demodulator only the modulation of the stronger signal will be heard (function $\Delta\omega_1 \cos p\mu_1 t$). The modulation of the weaker signal is lost in intermodulation products caused by multiples of both the modulation frequencies and the carrier frequencies.

If the frequency difference of the carriers is large enough these products will not fall within the AF passband of the demodulator output filter. The modulation of the weaker signal is only perceptible if $\omega_1 = \omega_2$. This gives a theoretical explanation of the capture effect, which can be found in many publications [2]-[6]. *In practice, however, Foster-Seeley and ratio demodulators do not show such an extreme capture effect.* It is possible to cancel the influence of the capture effect by limiting the bandwidth of the demodulator. This is explained by taking into account the practical physical behavior of the demodulator.

To illustrate of the problem better, the frequency spectrum of the limiter output signal will be observed in the next section.

IV. THE FREQUENCY SPECTRUM OF THE LIMITER OUTPUT SIGNAL

By (10) the phase angle of the resultant R was given as

$$\Psi = \phi_1 - \sum_{n=1}^{\infty} \frac{(-A)^n}{n} \sin n\phi.$$

The amplitude information of the signal is removed by the limiter. The output signal can also be described as

$$u(t) = \mathrm{Re}\,[Ce^{j\Psi}] = \mathrm{Re}\left[Ce^{j\phi_1} \exp\left(j\left\{-\sum_{n=1}^{\infty} \frac{(-A)^n}{n} \sin n\phi\right\}\right)\right] \qquad (22)$$

in which C is constant, defined by the limiter. The spectrum of

$$\exp\left(j\left\{-\sum_{n=1}^{\infty} \frac{(-A)^n}{n} \sin n\phi\right\}\right)$$

can be determined by extension in a Taylor's series as follows:

$$\exp\left(j\left\{-\sum_{n=1}^{\infty} \frac{(-A)^n}{n} \sin n\phi\right\}\right) = S_0 + S_{+1}e^{j\phi} + S_{-1}e^{-j\phi} + S_{+2}e^{j2\phi} + S_{-2}e^{-j2\phi} \text{ etc.} \qquad (23)$$

So

$$u(t) = \mathrm{Re}\, C[S_0 e^{j\phi_1} + S_1 e^{j\phi_2} + S_{-1}e^{j(2\phi_1 - \phi_2)} + S_2 e^{j(2\phi_2 - \phi_1)} + S_{-2}e^{j(3\phi_1 - 2\phi_2)}] \text{ etc.} \qquad (24)$$

in which

$$S_0 = 1 - \frac{A^2}{4} + \cdots \text{ etc.} \quad S_2 = -\left(\frac{A^2}{8} + \cdots\right) \text{ etc.}$$

$$S_1 = \frac{A}{2} + \frac{A^3}{16} + \cdots \text{ etc.}$$

$$S_{-1} = -\left(\frac{A}{2} - \frac{A^3}{16} + \cdots\right) \text{ etc.}$$

As an illustration, the spectral amplitude pattern for $A = 0.6$ in a quasi-stationary approach is given in Fig. 3 for $\Omega_2 > \Omega_1$. The amplitudes of the spectral components as functions of A are given in Fig. 4. It is remarkable that there is a relative attenuation of the weaker signal. This can be seen in the example, given in Fig. 3.

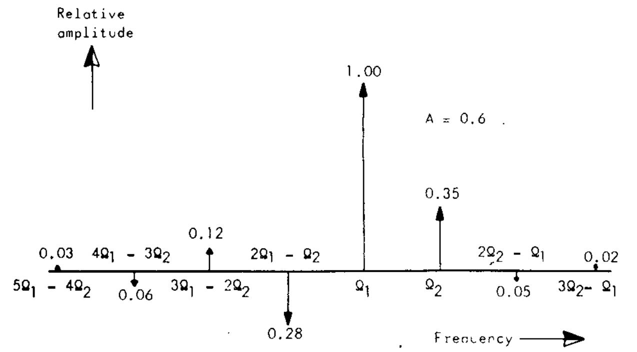

Fig. 3. Frequency spectrum of the limiter output signal of two signals is supplied to its input with frequencies Ω_1 and Ω_2. Amplitude ratio of the two signals $A_2/A_1 = A = 0.6$.

Normalized on the amplitude $A_1 = 1$, the amplitude A_2 at the limiter input is 0.6 and in the output spectrum 0.35. This could be expected looking at the phasor diagram at Fig. 1. The spectrum components have significant amplitudes only for $10 > A > 0.1$. For values of A between these limits the output spectrum is very complex. One should realize that a multiple of Ω_1 or Ω_2 means that the modulation indices of the signals are multiplied too, so new spectra are created.

The instantaneous Ω_1 and Ω_2 can be replaced by central frequencies ω_1 and ω_2 with their coupled spectra if these are FM signals

The amplitude functions in Fig. 4 also can be found by measuring the output signals of a limiter with a spectrum analyzer.

The spectral amplitude pattern is very complex and does not lead to an understanding of the physical behavior of the demodulator.

A derivation of the capture effect will be given considering the output signal of the demodulator in the time domain in the following section.

V. DEMODULATION

Imagine an ideal FM demodulator in which it would be possible to detect each spectrum component separately, and to sum all AF output signals. Calculating the AF output of that demodulator, and taking into account the different phase angles and deviations, one finds the same curve for the AF output as a function of A as given in Fig. 2. This curve does not show any capture effect at all. So if it would be possible to construct such a demodulator, the capture effect should not appear. Experiments done with synchronous demodulators, in which it is possible to detect one spectrum component separately by means of correlation, did show that in this case also the capture effect did not exist.

In (10) the frequency of the resultant output signal of the limiter was given by

$$\Omega_r = \Omega_1 + \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi} (\Omega_2 - \Omega_1).$$

In Fig. 5

$$x = \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi} \qquad (25)$$

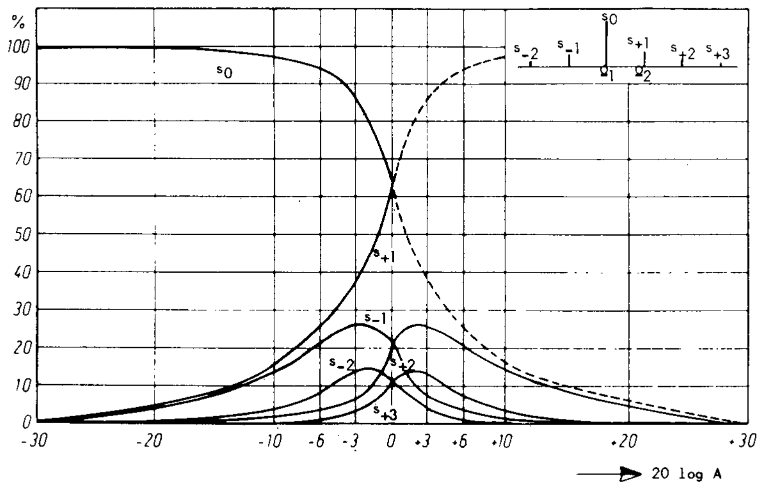

Fig. 4. Relative amplitudes of the spectral components of the limiter output signal when two signals are supplied to its input as a function of amplitude ratio $A_2/A_1 = A$, normalized on $A_1 = 100$ percent. Frequencies of the input signals are Ω_1 and Ω_2.

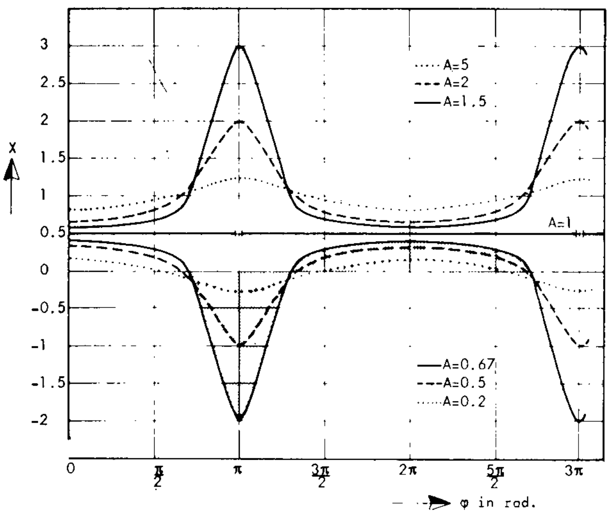

Fig. 5. The function $x = (A^2 + A\cos\phi)/(1 + A^2 + 2A\cos\phi)$ as a function of phase difference ϕ of the two input signals of the limiter. $A = A_2/A_1$ is the amplitude ratio of these two signals.

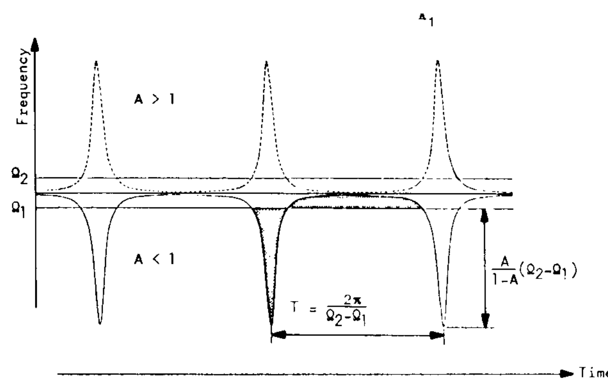

Fig. 6. Resultant frequency of the limiter output signal as a function of time when two signals with frequencies Ω_1 and Ω_2 are supplied to the limiter input. Amplitude ratio A of the signals may be <1 (drawn graph) or >1 (dotted graph), Ω_1 and Ω_2 instantaneously constant.

is sketched as a function $\Omega = (\Omega_2 - \Omega_1)t$ for some different values of A. When $A = 1$, there is a discontinuity at $\phi = \pi$, the phase of R suddenly changes from $+\pi/2$ to $-\pi/2$; at this moment a large change of the resultant frequency Ω_r will occur.

It is possible to determine the mean value of the function x by integration

$$I = \frac{1}{\pi}\int_0^{\pi} \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\, d\phi \tag{26}$$

which gives

$$I = \frac{1}{2} - \frac{1}{\pi}\arctan\frac{1-A}{1+A}\cdot p\Bigg|_{p=0}^{p=\infty} \tag{27}$$

if $A > 1$, then $I = 1$.

In this case, the mean frequency of the resultant is

$$\Omega_{r_{\text{mean}}} = \Omega_1 + (\Omega_2 - \Omega_1) = \Omega_2 \tag{28}$$

if $A < 1$, then $I = 0$. Now

$$\Omega_{r_{\text{mean}}} = \Omega_1. \tag{29}$$

In other words the mean frequency of the resultant is always the frequency of the stronger signal.

In Fig. 6 $\Omega_{r\,\text{mean}}$, Ω_1, and Ω_2 are presented as a function of time.

Figs. 5 and 6 convey the following information.

1) The output signal of the limiter which is fed to the demodulator has a mean frequency equal to the frequency of the stronger signal.

2) The actual frequency of the signal fed to the demodulator has peaks, which are asymmetrical with respect to the mean frequency.

3) The peaks always "point away" from the frequency of the weaker signal.

4) The peaks are sharper, the more A nears 1, and are infinitely high when A equals 1.

5) The peaks are smaller when the frequency difference between Ω_1 and Ω_2 is smaller.

6) The frequency of the peaks Ω_p is equal to the frequency difference between Ω_1 and Ω_2.

In Fig. 6, both frequencies Ω_1 and Ω_2 are supposed to be constant. When one signal is modulated, e.g.,

$$\Omega_2 = \omega_2 + \Delta\omega_2 \cos \mu_2 t \tag{13}$$

then the time intervals between the peaks vary with cos $\mu_1 t$, as

$$\Omega_p = \Omega_2 - \Omega_1 = \omega_2 - \omega_1 + \omega_2 \cos \mu_2 t. \tag{30}$$

In Figs. 7-10 some graphs are given for different as well as equal carrier frequencies. These pictures can be obtained on an oscilloscope when measuring before the low-pass filter in the demodulator.

If the output of the demodulator is proportional to Ω_r, the frequency of the input signal, then the output signal will have the same shape as Ω_r. When the output signal has the shape of Fig. 6, Fourier analysis shows that the dc component p_0 of the signal is

$$p_0 = \frac{1}{T} \int_{-T/2}^{+T/2} f(t) \cdot dt. \tag{31}$$

Integrating over the time interval T between two peaks, one will find that this integral resembles the integral (26), so that the dc component is proportional to the mean frequency of Ω_r, and therefore to the frequency of the stronger signal.

When the intervals between the peaks vary, the dc component will vary too, and a low-frequency component will originate. This is the case when one signal or both signals are frequency modulated.

In the preceding paragraphs the limited bandwidth of the receiver circuits has not been taken into account. Therefore capturing has been found to be absolute. In the following section a closer look at the physical limitations will be taken.

It is acknowledged that working with instantaneous values of frequency in frequency modulation has serious limitations. We, however, believe that these limitations do not restrict the validity of the present treatment.

VI. THE PUSH-PULL DEMODULATOR

A frequently used type of demodulator is the push-pull demodulator. In Fig. 11 the circuit is shown. For the bandpass filter the following equation can be calculated:

$$\frac{u_2}{u_1} = \frac{-jkQ_2\sqrt{L_2/L_1}}{1 + j\beta Q_2}. \tag{32}$$

This is the equation of a circle in which $\beta = 2\Delta\omega/\omega_0$ is the only variable. So β is proportional to the deviation $\Delta\omega$. In normal circuits

$$kQ_2\sqrt{L_2/L_1} \approx 2. \tag{33}$$

u_2 may be approximated by

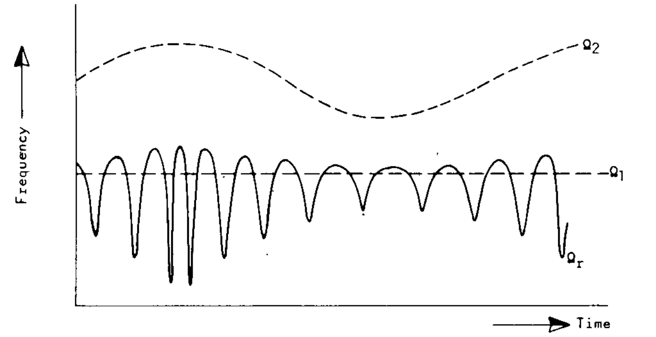

Fig. 7. Resultant frequency Ω_r of the limiter output signal if two signals are supplied to the limiter input. Signal a_1 (frequency Ω_1) is unmodulated, signal a_2 (frequency Ω_2) is FM. The carrier frequencies of the two signals are different. Amplitude $A_1 > A_2$. Ω_r varies along the value of Ω_1; the mean value of Ω_r is Ω_1. The peaks in Ω_r point away from the Ω_2 curve.

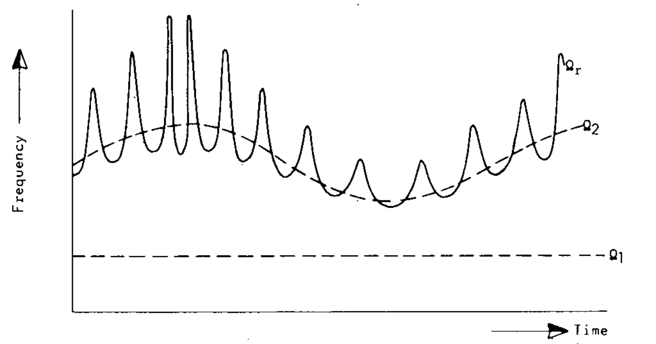

Fig. 8. Resultant frequency Ω_r of the limiter output signal if two signals are supplied to the limiter input. Signal a_1 (frequency Ω_1) is unmodulated, signal a_2 (frequency Ω_2) is FM. The carrier frequencies of the two signals are different. Amplitude $A_1 < A_2$. Ω_r varies along the value of Ω_2; the mean value of Ω_r is Ω_2. The peaks in Ω_r point away from the Ω_1 curve.

$$u_2 = \frac{-j2}{1 + j\beta Q} u_1 = \frac{2\beta Q}{1 + \beta^2 Q^2} u_1 - j \frac{2}{1 + \beta^2 Q^2} u_1. \tag{34}$$

Now the polar diagram can be constructed, as shown in Fig. 12. u_3 and u_4 are the sum voltages at the detector diodes.

The output signal of the demodulator is

$$|u_4| - |u_3| = |u_1| \cdot \frac{4\beta Q}{\sqrt{1 + \beta^2 Q^2}\{\sqrt{1 + (1 - \beta Q)^2} + \sqrt{1 + (1 + \beta Q^2}\}}. \tag{35}$$

u_1 is held at a constant amplitude by the limiter.

From this formula one can conclude that the demodulator operates linearly only for small frequency deviations, or at a low value for βQ. At large deviations the output of the demodulator is "attenuated" by a factor

$$K = 1/\sqrt{1 + \beta^2 Q^2}\{\sqrt{1 + (1 - \beta Q^2} + \sqrt{1 + (1 + \beta Q)^2}\}. \tag{36}$$

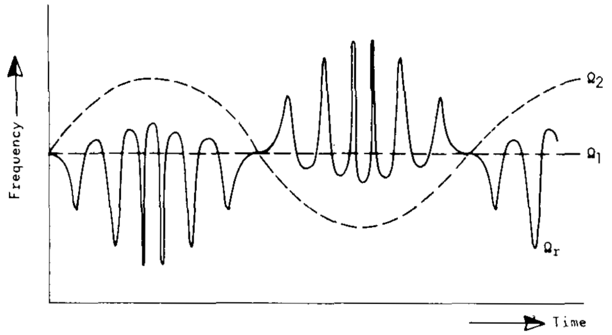

Fig. 9. Resultant frequency Ω_r of the limiter output signal if two signals are supplied to the limiter input. Signal a_1 (frequency Ω_1) is unmodulated, signal a_2 (frequency Ω_2) is FM. The carrier frequencies of the two signals are equal. Amplitude $A_1 > A_2$. Ω_r varies along the value of Ω_1; the mean value of Ω_r is Ω_1. The peaks in Ω_r point away from the Ω_2 curve.

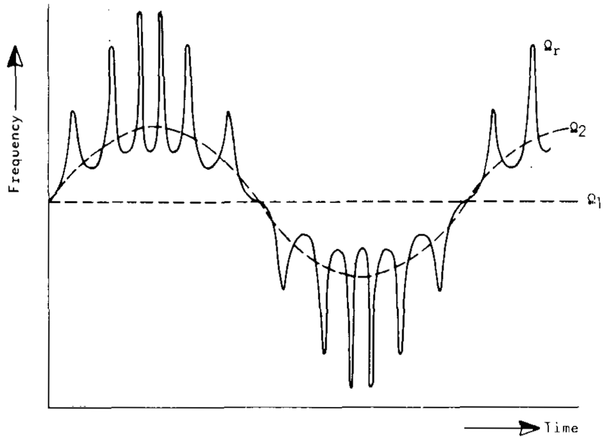

Fig. 10. Resultant frequency Ω_r of the limiter output signal if two signals are supplied to the limiter input. Signal a_1 (frequency Ω_1) is unmodulated, signal a_2 (frequency Ω_2) is FM. The carrier frequencies of the two signals are equal. Amplitude $A_1 < A_2$. Ω_r varies along the value of Ω_2; the mean value of Ω_r is Ω_2. The peaks in Ω_r point away from the Ω_1 curve.

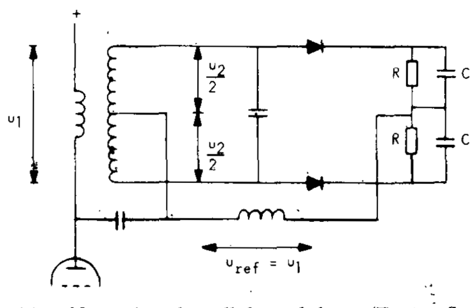

Fig. 11. Normal push-pull demodulator (Foster-Seely).

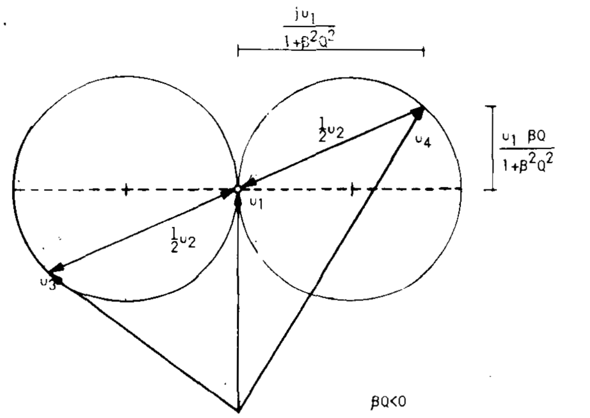

Fig. 12. Polar diagram for a push-pull demodulator. U_2 is the secondary voltage; U_1 is the reference voltage; U_3 and U_4 are the resultant voltages at the demodulator diodes.

This correction factor K is given in Fig. 13 as a function of βQ, normalized on the value of K for $\beta Q = 0$ $(= 0.35)$.

One can see that the attenuation is high at the moment the instantaneous frequency deviation is large. This results in a high attenuation of the peaks which appear in the frequency function given in Figs. 6–10. Figs. 14 and 15 are examples of photographs taken from an oscilloscope screen, when measuring before the low-pass filter in the demodulator. These photographs show this effect very clearly.

The corrected mean value of each peak is given by

$$I' = \frac{1}{\pi}\int_0^{\pi} K(\phi)\,\frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\,d\phi \qquad (37)$$

$$I' < 1 \quad \text{if} \quad A > 1$$

$$I' > 0 \quad \text{if} \quad A < 1.$$

Note that K is a function of ϕ as K is a function of the instantanteous frequency shift.

Therefore, the capture effect is attenuated and the modulation of the weaker signal is still perceptible if A does not have extreme low or high values. The instantaneous ouptut of the demodulator is a function of $\Omega_R - \Omega_1$, if Ω_1 equals the resonance frequency of the demodulator. If two signals with equal carrier frequency are at the input of the limiter, one unmodulated and the other one modulated

$$\Omega_1 = \omega_1 = \omega_2$$

$$\Omega_2 = \omega_2 + \Delta\omega_2 \cdot \cos\mu_2 t$$

in which ω_1, ω_2, and $\Delta\omega_2$ are constants, then the instantaneous output of the demodulator is proportional to

$$\Omega_r - \Omega_1 = K_{(\phi)}\,\frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\,(\Omega_2 - \Omega_1)$$

$$= K_{(\phi)}\,\frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\,\Delta\omega_2\cos\mu_2 t. \qquad (38)$$

The mean value of the amplitude of the AF output of the

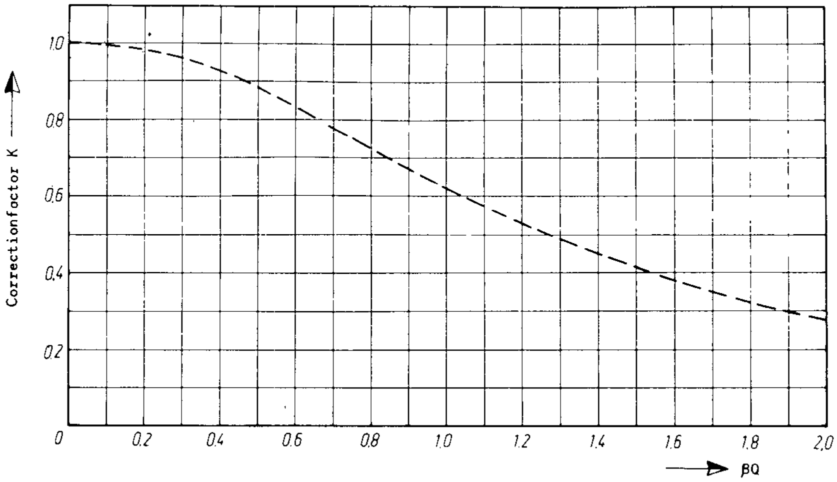

Fig. 13. Attenuation factor K for the output signal of a push-pull demodulator as a function of βQ.

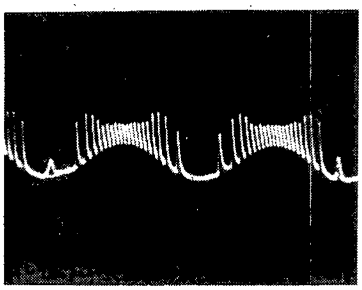

Fig. 14. Output signal of a demodulator before the low-pass filter if two signals a_1 and a_2 are supplied to the foregoing limiter input. Signal a_1 is not modulated, signal a_2 is modulated with 1000 Hz, deviation 10 kHz. Difference in carrier frequency is also 10 kHz. $20 \log A_2/A_1 = +1$ dB.

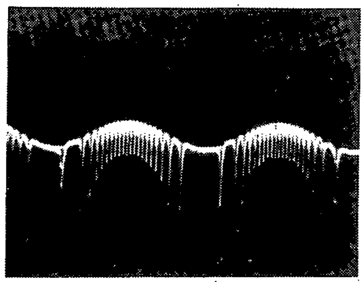

Fig. 15. Output signal of a demodulator before the low-pass filter if two signals a_1 and a_2 are supplied to the foregoing limiter input. Signal a_1 is not modulated, signal a_2 is modulated with 1000 Hz, deviation 10 kHz. Difference in carrier frequency is also 10 kHz. $20 \log A_2/A_1 = -1$ dB.

demodulator is now proportional

$$\frac{\Delta\omega 2}{\pi} \int_0^{\pi} K_{(\phi)} \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi} \cdot d\phi.$$

Calculate the value of I' as follows:

$$\frac{1}{\pi} \int_0^{\pi} K_{(\phi)} \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}\, d\phi \tag{39}$$

where

$$K_{(\phi)} = \frac{2\sqrt{2}}{\sqrt{1 + \left(\frac{2\Delta f}{B}\right)^2} \left\{ \sqrt{1 + \left(1 - \frac{2\Delta f}{B}\right)^2} + \sqrt{1 + \left(1 + \frac{2\Delta f}{B}\right)^2} \right\}}, \qquad \frac{2\Delta f}{B} = \beta Q \tag{40}$$

$$2\Delta f = \frac{\Delta\omega^2}{\pi} \frac{A^2 + A\cos\phi}{1 + A^2 + 2A\cos\phi}. \tag{41}$$

In Fig. 16 two computer calculated curves b and c are presented, giving the relative AF output amplitude of the demodulator as a function of A. Curve b is calculated for a demodulator bandwidth of $B = 300$ kHz. Curve a is calculated for a demodulator bandwidth of $\Delta f = 50$ kHz. For curve c these values are $B = 1000$ kHz and $\Delta f = 50$ kHz.

In the same figure curve d presents the measured output of a Foster-Seeley demodulator with two signals with the same carrier frequency at the limiter input; one of these was modulated with a maximum deviation of 50 kHz at a modulation frequency of 1000 Hz. The carrier frequency was 10 MHz.

The band filter of the demodulator had a bandwidth of 300 kHz; for this signal

$$\frac{2\Delta f}{B} = \frac{2.50 \cdot 10^3}{3 \cdot 10^5} = 0.33.$$

The AF output was measured with a selective voltmeter (wave analyzer). Curve a in Fig. 16 is the theoretical AF output at 100-percent capturing.

Comparing curves b, c, and d, it is evident that equation gives a good approximation to the output of the demodulator. It shows the capture-effect as it is present at demodulation, using a demodulator with a limited bandwidth, as is normally used in FM receivers.

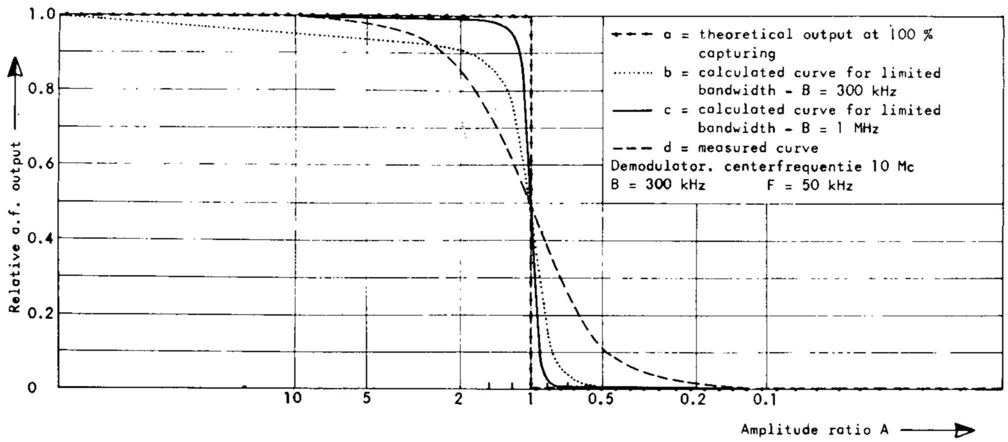

Fig. 16. Relative audio frequency output of a demodulator with two signals at the limiter input as a function of the amplitude ratio A of these two signals.

CONCLUSIONS

By the foregoing we showed that it is possible to suppress the capture effect by limiting the bandwidth of a demodulator. Of course this can give distortion, but it may be possible to choose a bandwidth giving a low distortion at a deviation chosen, and yet giving a good suppression of the capture effect by attenuation of the deviation peaks which occur at interference.

According to Fig. 16, curve b, for $A = 0.8$ the suppression of the weaker signal is 20 dB. At full capturing this signal would not be audible at all. This may be important in satellite communication, where, normally, hard limiters are used in the repeaters, so the capture effect can be very strong.

Demodulators with large bandwidths will show strong capture effect. Demodulators with small bandwidths show less capture effect. Measurements done on different types of demodulators confirm this. Counting demodulators do have a large bandwidth and show strong capture effect.

A very small bandwidth can be achieved by using a synchronous demodulator. Experiments using a synchronous demodulator show that a weak capture effect can be obtained.

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