# Comparison of Demodulation Techniques for MSK

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*Abstract.* For MSK, three demodulators are compared. The first demodulation algorithm, partially coherent demodulation, is based on a classical matched filter approach combined with feedforward phase synchronization, whereas the second algorithm, block demodulation, is based on minimizing a distance measure based on the symbol vector trial and the observed differential phase vector. The third algorithm is based on the same distance measure, however the minimization is carried out using the viterbi algorithm. We provide a derivation of the second algorithm. It is shown that the first approach is superior both in performance and computational complexity. The first algorithm also exhibits the best robustness properties in the case of signal impairments.

## 1 Introduction

MSK IS a modulation technique widely known to the research community. Although the difficulty of synchronizing MSK signals due to their stronger intersymbol interference (ISI) effects difficulties in finding optimum receivers, MSK *is* attractive in all cases where nonlinearities in the analogue signal path enforce using a modulation technique with minimal amplitude fluctuations.

This is the case especially when either the application domain has not yet been explored extensively for finding (almost) linear analogue receivers and transmitters or when receiver nonlinearities are accepted for reducing implementation costs. However, in all cases, one is still interested in finding optimum demodulators with an acceptably simple implementation.

In this contribution, we investigate three modulation methods for MSK, which are a partially coherent MSK demodulator (PC) and an incoherent block demodulation approach (BD) [1] and thirdly differential phase demodulation using the Viterbi algorithm (VA). In [2], a similar scheme is employed using a limiter discriminator, in [3] Viterbi sequence estimation is used based on the baseband signal. In here, we will focus on Viterbi detection considering the differential phase.

After briefly giving necessary definitions and describing the three demodulators mentioned already we compare the demodulators in terms of their computational efficiency and their performance. We derive the BD approach from coherent demodulation. The VA demodulator is motivated by observation of the MF outputs.

In any case, our analysis is limited to the effects of carrier phase distortion in a non (or only slowly) flat fading channel. Timing recovery is not discussed in here, but has been discussed previously in [4,5,6] and others.

# 2 Signal Models

## 2.1 MSK

MSK modulation can be described in two ways; firstly, we look at the signal phase only, in this case the *modulated signal* for $t \in [\,KT,\,(K+1)T\,]$ (symbol $K$ is transmitted) and $\tau = t - KT$ is described as

$$s(t) \quad = \quad c \cdot \exp\left[ j \left( \sum_{k=0}^{K-1} a_k \frac{\pi}{2} + a_K \frac{\pi}{2} \frac{\tau}{T} \right) \right] \mathrm{e}^{j\Phi_0} \tag{1}$$

where $a_k = \pm 1$ represent the data bits.

Quantities having an impact on the signal phase may be gathered in matrices: Let the signal phase vector $[\ldots \arg(s(KT)) \ldots]^T$ be denoted as $\boldsymbol{\Psi} + \Phi_0$. The vector $\boldsymbol{\Psi}$ is expressed in terms of the symbol vector $\boldsymbol{a}$ as

$$\boldsymbol{\Psi} \;=\; \frac{\pi}{2} \underbrace{\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \cdots & 1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ 0 & \cdots & 0 & 0 \end{pmatrix}}_{\mathbf{W}} \boldsymbol{a} \bmod 2\pi \qquad \begin{array}{l} \boldsymbol{a} \text{ is of size } L \times 1, \\ \mathbf{W} \text{ of size} (L+1) \times L. \end{array} \tag{2}$$

Let us introduce the modulation pulse with energy $E_B$ and duration $2T$

$$g(t) = \left\{ \begin{array}{ll} \frac{\sqrt{E_B}}{T} \cos \frac{\pi t}{2T} & t \in [-T, T] \\ 0 & \text{elsewhere} \end{array} \right\}. \tag{3}$$

Using Eulers formula and performing some lengthy computation, we achieve a convenient baseband representation of the MSK signal for all $t$ [7].

$$s(t) = c \cdot \mathrm{e}^{j\Phi_0} \sum_{K=0}^{K=N-1} b_K j^K g(t - KT), \tag{4}$$

where $b_K = \prod_{k=0}^{K-1} a_k$ for $K > 0$ and $b_K = \prod_{k=K-1}^{0} a_k$ for $K \leq 0$. For our analysis we assume a white additive gaussian noise channel described by a double sided noise spectral density $N_0$. The Phase offset $\Phi_0$ is initially assumed constant, but during simulations, we investigated a dynamic phase, as well. For simplicity, a vector consisting of elements $\Phi_0$, ie $[\ldots \Phi_0 \ldots]^T$ also is denoted with $\Phi_0$.

Hence both $I$ and $Q$ channel are modulated alternatingly with the pulse $g(t)$ leading to a signal with constant amplitude. The received signal is thus expressed as

$$r(t) = s(t) + n'(t) \tag{5}$$

In the following, let $c = 1$.

## 2.2 The Matched Filter Approach

The optimum approach to demodulate MSK is to employ a matched filter (MF). Filtering any pulse $g(t)$ with the matched filter $g(-t)$ will lead to the MF outputs

$$m'(t) = \int_{\max(0,t)}^{\min(2T,2T+t)} g(\tau')\cdot g(t-\tau')d\tau' = \left(\left(1-\frac{|t|}{2T}\right)\cos\frac{\pi t}{2T} + \frac{1}{\pi}\sin\frac{\pi|t|}{2T}\right) E_B \tag{6}$$

when $t \in [-2T, 2T]$ and $m'(t) = 0$ elsewhere. Hence $b_K m'(\pm T) = E_B b_K/\pi$, $b_K m'(0) = E_B b_K$ and $b_K m'(kT) = 0, |k| > 1$. The output of the matched filter applied to the *complex* noisy baseband signal (5) thus has the form

$$m(t) = \mathrm{e}^{j\Phi_0} \sum_{K=0}^{K=N-1} b_K j^K m'(t-KT) + n(t) \tag{7}$$

using the time limitations of $m'(\cdot)$, we obtain at $t = KT$

$$m(KT) = E_B \mathrm{e}^{j\Phi_0} b_{K-1} j^K \left(\frac{1}{\pi} + j a_{K-1} - \frac{1}{\pi} a_{K-1} a_K\right) + n(KT), \quad K > 0. \tag{8}$$

which is *not* ISI - free. Figure 1 depicts possible values of $m(KT)$. Detection consists of minimizing the norm of the vector

$$\mathbf{e}_{MF} = [m(kT) - E_B \mathrm{e}^{j\hat{\Phi}_0} j^{k-1}(\frac{1}{\pi}\hat{b}_{k-1} + j\hat{b}_k - \frac{1}{\pi}\hat{b}_{k+1})], k = 0 \ldots N-1 \tag{9}$$

for a given MF output vector and requires a phase estimation.

For even $K$ the actual symbol (A,B,C,J,K,L in figure 1) is transmitted on the $Q$ channel. If the previous and the next symbol will be mapped both on the positive $I$ axis, an ISI of $2E_B/\pi$ is observed at the correct sampling instant. Given that the adjacent symbols are mapped on positive and negative side of the In-Phase axis, no ISI is observed. However, as ISI and symbols of interest always are transmitted on separate axes (either I or Q), the ISI always can be cancelled by ignoring either real or imaginary part of the MF output. For even $K$, the real part of the MF output has to be removed.

If the signal phase is known, removal of the ISI will thus cause no problem. However, if the phase is not known, ISI will affect demodulation.

The noise $n(KT)$ in this case is colored but gaussian. Using (8), the (optimum) SNR is

$$\gamma_{MF} = E_B/N_0 \tag{10}$$

## 2.3 The Differential Phase Approach

Suboptimum receivers may be built based on detecting the sign of the $L$ elements of the differential phase vector of the narrowband filtered ($h(kT)$, effective Bandwidth $B_h$) MSK signal, this technique is of an entirely different

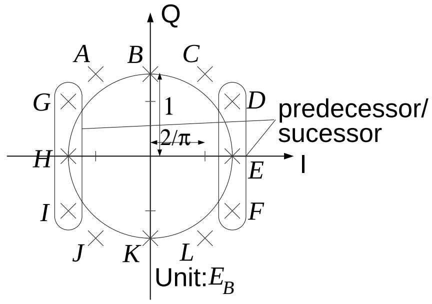


**Figure 1** MF outputs for MSK signal for $\Phi_0 = 0$.

flavor and has its roots in the theory of suboptimum CPM receivers applicaple to any CPM signal [7]. In addition to the advantage of extendability to other modulation formats, a phase synchronization unit is avoided.

Focusing on the differential phase hence provides the receiver with observations

$$\Phi(KT) = \arg\left((s(KT)+n(KT)) * h(kT)\right) + \Phi_0 \quad (11)$$

$$\boldsymbol{\Phi} =: \underbrace{\begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & 1 & -1 & 0 \\ 0 & \cdots & 0 & 1 & -1 \end{pmatrix}}_{\mathbf{T}} \begin{bmatrix} \vdots \\ \Phi(lT) \\ \vdots \end{bmatrix} + \mathbf{T}\mathbf{n}_\phi. \quad (12)$$

The (phase) noise $n_\phi$ perturbing the observations is neither gaussian nor white (due to the filtering with $h(KT)$ in the $I/Q$–domain). Focusing on the differential phase, the noise model has to be modified to the consideration of the *phase* perturbation, which is discussed eg in [8] [1] [9]. For our purposes, it is accurate enough to model the (mathematically complex) [8] phase perturbation $n_\phi(KT)$ as gaussian [5] with a variance of $1/2\gamma_h$, where $\gamma_h = E_B/(N_0 B_h T)$ is the SNR per bit. $\mathbf{T}$ is of size $L \times (L+1)$.

Employing the matched filter as $h(kT)$ will create strong ISI. When employing the matched filter only, differential phase values of 'transitions' as depicted in figure 1 are summarized in table 1. Hence, six different phase values

| Trans. | Phase | Trans. | Phase |
|---|---|---|---|
| G-A | $\arg(-1+j2/\pi)(-2/\pi-j)$ | D-C | $\arg(-1+j2/\pi)(-2/\pi-j)$ |
| H-A | $-\arg(2/\pi+j)$ | E-C | $\arg(2/\pi+j)$ |
| G-B | $-\arg(2/\pi+j)$ | D-B | $\arg(2/\pi+j)$ |
| H-B | $-\pi/2$ | E-B | $\pi/2$ |

**Table 1** Differential phase observed at the output of the matched filter.

can be observed. Transitions to the terminal points A,B and C which are not mentioned cannot occur. Transitions E-C and D-B, for example, lead to the same phase difference values, since exchanging preceding and succeeding symbols of the actual symbol does not change its accumulated ISI. When transmitting a sequence of symbols, the sequence of differential phase values is best described using a trellis diagram, whose exploitation for data detection using the Viterbi algorithm is described in section 3.3.

Using a filter with a larger bandwidth, ISI in the differential phase will be smaller, however, the same splitting in six (or more) differential phase values can be observed eg as described for a limiter discriminator in [2].

In case a *mismatched* filter is employed, the filter impulse response $h(kT)$ is *not* optimized with respect to the modulation pulse but under consideration of the BER given the suboptimum demodulation method [10], [4], possibly including the effect of frequency offsets [11] [12], and the sign of $\phi(KT)$ is used for detection.

# 3 Demodulation

## 3.1 Partially Coherent Demodulation (PC)

The first approach to demodulation is to provide the receiver with a phase reference and to detect the data from the MF outputs. This is not a new approach, but is developed for providing a basis to understand the goal of the paper. Minimizing $\mathbf{e}_{MF}$ from eqn. (9) is achieved by computing a phase reference and performing serial detection:

**Phase Reference.** We firstly discuss computing the phase reference. *Feed-forward* phase synchronization is best accomplished *before* the MF, as in this case ISI does not perturb the signal to the same extent. The price for pre-MF phase synchronization is to increase noise and to provide a second filter stage, on the other hand, phase dynamics may easier be compensated. To remove the modulation, we neglect any ISI caused by the prefilter, ie we assume

$$r(lT) \approx r(lT) * h(lT). \tag{13}$$

It has to be considered that at any time $KT$ the MSK signal $r(KT)$ may assume two values which are located on the real axis at odd sampling instants and on the imaginary axis at even sampling instants. The twofold ambiguity is removed by squaring the samples[1], alteration between I and Q channel by multiplying any second sample with $j$. We obtain the phase estimate [14]

$$\hat{\Phi}_0 = \frac{1}{2} \arg \sum_{l=0}^{L-1} (-1)^l (r(lT) * h(lT))^2 \tag{14}$$

We have to compare the performance of this estimator to the CRLB [15]

$$\boldsymbol{Var}\hat{\Phi}_0 \geq \frac{1}{2L\gamma_{MF}}. \tag{15}$$

[1] Other nonlinearities may be of interest [13]

Since the noise after having passed through the prefilter, only, has a higher noise variance, an 'optimum' estimator for this filter may only achieve a variance

$$\boldsymbol{Var}\,\hat{\Phi}_0 \geq \frac{1}{2L\gamma_h} \tag{16}$$

Simulation results are displayed in figure 2 for estimator lengths $L$ of relevance. The phase estimate $\hat{\Phi}_0$ is used to adjust the received signal's phase and ensures the first symbol to be real (but ambiguous due to the twofold phase ambiguity). As the performance bound (16) is asymptotically reached, the use of approximation (13) to obtain the estimator is justified.

Phase synchronization is performed using a gliding window.

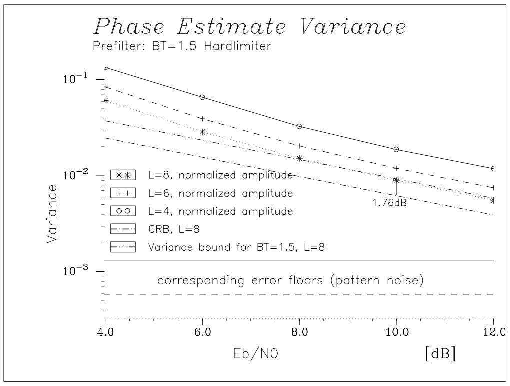


**Figure 2** Performance of feedforward MSK phase synchronizer with hardlimiting, normalized bandwidth $BT = 1.5$.

**Serial Detection.** It is known [7] [16] that coherent detection of MSK is best performed in the bandpass domain choosing an intermediate frequency $f_D = 1/4T$. In brief, the idea of this detection approach is to introduce zeros in the MF outputs such that the ISI between $I$ and $Q$ channel is removed.

A suitable bandpass transform is

$$s_{BP}(KT) = \mathrm{Re}\left(\exp\left(\frac{-j2\pi(K+1)}{4}\right)\cdot m(KT)\cdot e^{-j\hat{\Phi}_0}\right) \tag{17}$$

and assuming ideal phase recovery leads to

$$m_{BP}(KT) \quad = \quad E_B \cos\left(\frac{2\pi(K+1)}{4}\right) b_K + E_B \sin\left(\frac{2\pi(K+1)}{4}\right) b_K + \tilde{n}(KT) \tag{18}$$

Each of the cos (sin) terms vanishes in even (odd) time instants and thus the sequence

$$m_{BP}(KT) = E_B b_K + \tilde{n}(KT) \tag{19}$$

is obtained. Differential decoding produces data estimates $\hat{a}_K$. In fact, the real and imaginary part of the MF outputs are multiplied alternatingly with zeros and $\pm 1$ which removes the real or imaginary part of the MF output and hence leads to ISI free reception. Of course, phase errors $\Phi_0 - \hat{\Phi}_0$ will introduce ISI again. It has been shown that serial detection is less sensitive to phase errors than detection in the lowpass domain [17].

Due to the differential decoding procedure, any data bit output is based on *two* decisions, the error performance for large $L$ is given by [7,12]

$$p_{e,PC} = \operatorname{erfc}\sqrt{\frac{E_B}{N_0}}. \tag{20}$$

## 3.2 Block Demodulation (BD)

We consider a vector of $L+1$ phase values $\phi(KT)$ from (11) and then approximate the rule $\min_{\hat{b}} ||\mathbf{e}_{MF}||$ by observing the signal phase and neglecting ISI (which is commonly justified when not using the matched filter but a function $h$ with a larger bandwidth.).

$$\mathbf{e}_\phi = [\boldsymbol{\Psi} + \Phi_0 - \hat{\boldsymbol{\Psi}} - \hat{\Phi}_0] + \mathbf{n}_\phi \tag{21}$$

This metric will for high SNR be minimal for the same $\hat{b}_k$ as the original metric $\mathbf{e}_{MF}$. As before, we assume $\mathbf{e}_\phi$ gaussian and thus have to minimize $\mathbf{e}_\phi^T \left(\boldsymbol{Var}\mathbf{e}_\phi\right)^{-1} \mathbf{e}_\phi$ for approximating ML detection and maximizing the probability density of $\mathbf{e}_\phi$ conditioned on the symbol trials. Note that $\mathbf{e}_\phi$ is of size $(L+1)\times 1$.

How does the phase offset estimation error $\Phi_0 - \hat{\Phi}_0$ affect the statistics of $\mathbf{e}_\phi$? We assume an efficient estimate $\hat{\Phi}_0$ whose *effect* on the statistics is modeled by averaging over all noise samples belonging to the estimator window of length $L+1$. (Note that the CRB (15) may be interpreted in this way, as well.) The effect of introducing the phase estimate on the statistics of $\mathbf{e}_\phi$ is thus modeled by

$$\mathbf{e}_\phi = [\boldsymbol{\Psi} - \hat{\boldsymbol{\Psi}}] + \mathbf{n}_\phi + \underbrace{\frac{1}{(L+1)} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \sum_{i=0}^{L} n_{i,\phi}}_{\Phi_0 - \hat{\Phi}_0}. \tag{22}$$

After having removed the explicit occurrence of the phase offset,we can transform the metric into the differential phases using the transformation matrix $\mathbf{T}$ sized $L \times (L+1)$.

$$\mathbf{e}_{\Delta\phi} = \mathbf{T}\mathbf{e}_\phi \tag{23}$$

$\mathbf{e}_{\Delta\phi}$ is the vector whose probability conditioned on the symbol trials is to be maximized when detecting symbols using the differential phase.

Note that $\mathbf{T}\cdot\mathbf{W} = \mathbf{I}_{L\times L}$, the identity matrix (sized $L\times L$), and consequently

$$\mathbf{T}\,\hat{\boldsymbol{\Psi}} = \hat{\boldsymbol{a}}\,\frac{\pi}{2}. \tag{24}$$

Assuming $\mathbf{n}_\phi$ to be white gaussian noise, the noise correlation $\mathbf{C}$ of $\mathbf{e}_{\Delta\phi}$ is given by

$$\mathbf{C} := \sigma^2 \mathbf{T} \left( I_{L+1 \times L+1} + \frac{1}{L+1} \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{pmatrix} \right) \mathbf{T}^T \quad (25)$$

$$= 2\sigma^2 \begin{pmatrix} 1 & -0.5 & 0 & \cdots & 0 \\ -0.5 & 1 & -0.5 & \ddots & \vdots \\ 0 & -0.5 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -0.5 \\ 0 & \cdots & 0 & -0.5 & 1 \end{pmatrix} \quad (26)$$

which is the correlation matrix used in [1]. We have thus shown that neglecting ISI and considering an efficient phase estimate $\hat{\Phi}_0$ based on $L+1$ observations leads to the metric used in [1] when looking at the differential phase.

Providing a receiver with $L+1$ phase observations $\phi(KT)$ and an efficient phase estimate based on these observations is thus equivalent to providing $L$ succeeding phase difference observations $\Delta\phi = \phi(KT) - \phi(KT-T)$.

Symbol detection is based on maximizing the conditional probability of the occurrence of a symbol vector, ie minimizing $\mathbf{e}_{\Delta\phi}^T \mathbf{C}^{-1} \mathbf{e}_{\Delta\phi}$ by varying $\hat{\boldsymbol{a}}$ .

Thus we have understood that BD and PC implicitly perform the same functions (phase estimation and ML symbol detection), but in different signal spaces. BD, however, is based on several approximations.

We assess the effect of making these approximations using simulations.

As BD requires an exhaustive search [1], we seek for a means to reduce computing effort. Errors in differential detection of MSK are likely to occur when the phase difference between two successive symbols is close to zero or $\pm\pi$, either caused by noise or intersymbol interferences. On the contrary phase differences close to $\pm\pi/2$ may be considered as more reliable. Taking the distance measure $\Delta\phi - \pi/2$ or $\Delta\phi + \pi/2$ as a reliability estimate the $L_R$ most reliable phase differences may be used to detect the corresponding symbol by conventional DMSK threshold detection. Then, only the remaining $L - L_R$ symbols in $\boldsymbol{\Phi}$ have to be detected with the block demodulation procedure. Note, that now the evaluation of the detection metric and the minimization search has to be performed only for a reduced set of possible data patterns. Choosing $L_R = 2$ seems possible without performance degradation.

However, a further reduction of computational complexity can be achieved using the Viterbi algorithm.

## 3.3 Viterbi detection (VA)

The representation of the symbol transitions in a trellis is exploited. Depending on the two previous symbols, there exist four states in the trellis. Any state may be expanded with any new symbol leading to the trellis depicted in figure 3. A suitable metric for the minimum–distance path is (for

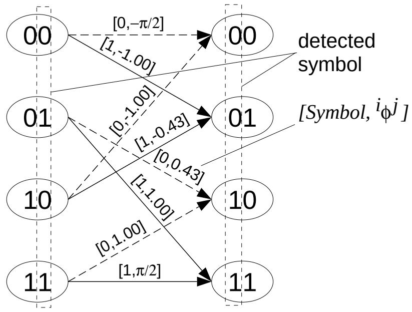


**Figure 3** Trellis of observed phase differences

gaussian noise) the Euclidean distance. In [9], a different metric was used which, however, does at least for moderate to high SNR not differ too much from the Euclidean distance. Simulation results confirmed the similar performance using either metric. Let the path metrics be described by $\Gamma_k^{1\ldots4}$ and the correct observations be denoted as ${}^{0,1}\phi_k^{1\ldots4}$ leading to the metric increments ${}^{i}\lambda^{k}$. Now the minimum path metric is given by the well known add–compare–select operations of the VA.

$$
\begin{aligned}
{}^{i}\Gamma_{k+1}^{j} &= \Gamma_k^i +{}^{i}\lambda_k^j && (27)\\
{}^{i}\lambda_k^j &= \left(\Delta\phi_k -{}^{i}\phi^j\right)^2 && (28)\\
\Gamma_{k+1}^{j} &= \min_i({}^{i}\Gamma_{k+1}^{j}) && (29)
\end{aligned}
$$

We expect some degradations as the increments of the paths are not independent and noise is not gaussian.

# 4 Computational Complexity

We consider any operation following the IF filter and divide into multiplications and additions, where all operations are scaled with the window length and normalized to the processing of $L$ symbols, respectively. We assume an oversampling rate of $4/T$ for computing the MF outputs. Both algorithms require a phase computation once per symbol.

From table 2 it is seen that PC demodulation requires less computations than BD when $L > 3$. Memory requirements for all algorithms are low. For PC and VA, the computing effort is independent of the observation interval. PC always requires less computations than VA.

| Operation | PC | | BD | | VA | |
|---|---|---|---|---|---|---|
| | Mult. | Add. | Mult. | Add. | Mult. | Add. |
| Diff. Phase | | | | $L$ | | 1 |
| Phase Synch. | $9L$ | $7L$ | | | | |
| Diff. Comp. | | | | $2L-2$ | | |
| Metric Comp. | | | $4(L-1)L-2$ | $2^{L-3}L(L+1)$ | 20 | 8 |
| M. Filt. | $16L$ | $16L$ | $16L$ | $16L$ | 16 | 16 |
| BP Transf. | $L$ | | | | | |
| **Σ/Symb.** | 26 | 23 | $4L+12-2/L$ | $2^{L-3}(L+1)+$ +19-2/$L$ | 36 | 25 |
| $L$=3,5,7 | | | 23,32,40 | 22,43,126 | | |
| Bit Oper. | | | | | | |
| Diff. Dec. | | $2L$ | | | | |
| Sign | | $2L$ | | | | |
| RP-conv. | | $L$ | | $L$ | | 1 |

**Table 2** Comparison of Computational Complexity. For BD, $L_R = 2$ was chosen. For VA, Operations / Symbol are given.

# 5 Signal Impairments

To compare the demodulation algorithms we look at the optimum synchronization scenario as well as at signal impairments, ie a residual frequency offset and random phase noise.

A frequency offset firstly shifts the signal spectra away from the zero frequency, hence the IF filter causes an asymmetric distortion of the signal spectrum leading to a degradation. For PC, the variance of the phase estimate is increased. For BD, the metric computation becomes biased. In general one may assume that the gliding window filter of PC will be able to cope better with frequency errors, as some sort of carrier tracking is established.

Finally, we investigated the influence of a hardlimiter in the signal path. The signal's amplitude was fixed after having passed the IF filter. Hence, in any case a further degradation was observed due to the fact that noise now becomes non–gaussian and secondly the IF filter introduced amplitude fluctuations.

# 6 Simulated Results

In figure 4, it can be seen that BD almost achieves the ideal performance for $L > 3$. PC achieves optimal performance for $L > 5$. VA also does not degrade for a static channel. The survivior depth proved as an uncritical parameter.

In case of a static frequency offset, BD is most sensitive, as seen in fig 4, right. Clearly this depends on the implicit assumption of a constant phase during the observation interval, whereas PC performs a tracking of the current phase and hence is more robust. On the other hand, VA is most sensitive against a random phase walk. Due to the slow change of the phase, a completely incorrect path may be selected in the trellis and cause many bit errors.

In figure 5, right, the degradations at 10dB $E_B/N_0$ are summarized (ie the ideal demodulator achieves the respective performance at $\Delta$dB less $E_B/N_0$). It is seen that PC is the most robust demodulation method.

# 7 Conclusions

We have discussed three demodulation schemes for MSK. Partially coherent demodulation is based on a feedforward phase estimator and serial demodulation of the MSK matched filter outputs. Block demodulation decides upon symbols providing the minimum–distance differential phase trajectory. Viterbi demodulation exploits the ISI of the phase differences produced by the matched filter. We derived block demodulation from partially coherent demodulation. It was shown that partially coherent demodulation for MSK is less complex and exhibits a better performance and robustness than block demodulation and also than Viterbi demodulation. In cases, where low computational complexity is desired, but an MF receiver cannot be realized, Viterbi detection will be an interesting alternative.

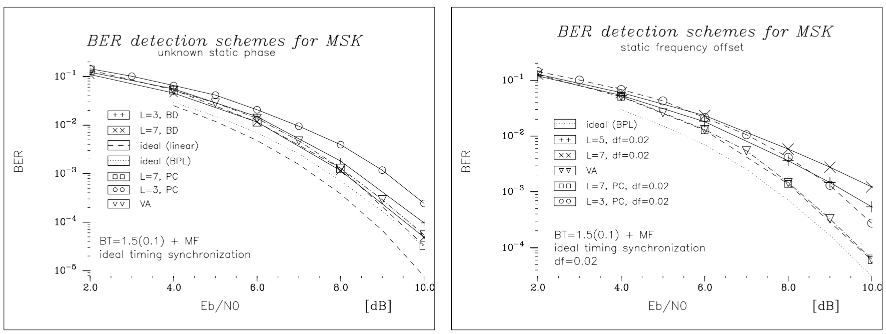


**Figure 4** The detection schemes for a static channel and a frequency offset of $dfT = 0.02$.

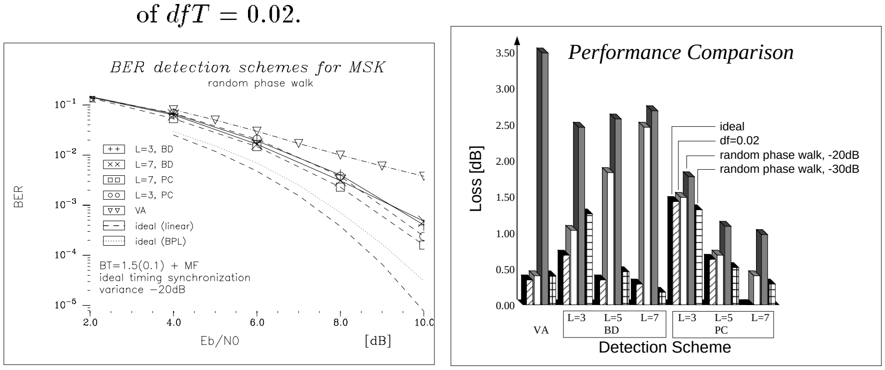


**Figure 5** The detection schemes for a random phase walk with a variance of -20dB. Losses at 10dB.

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